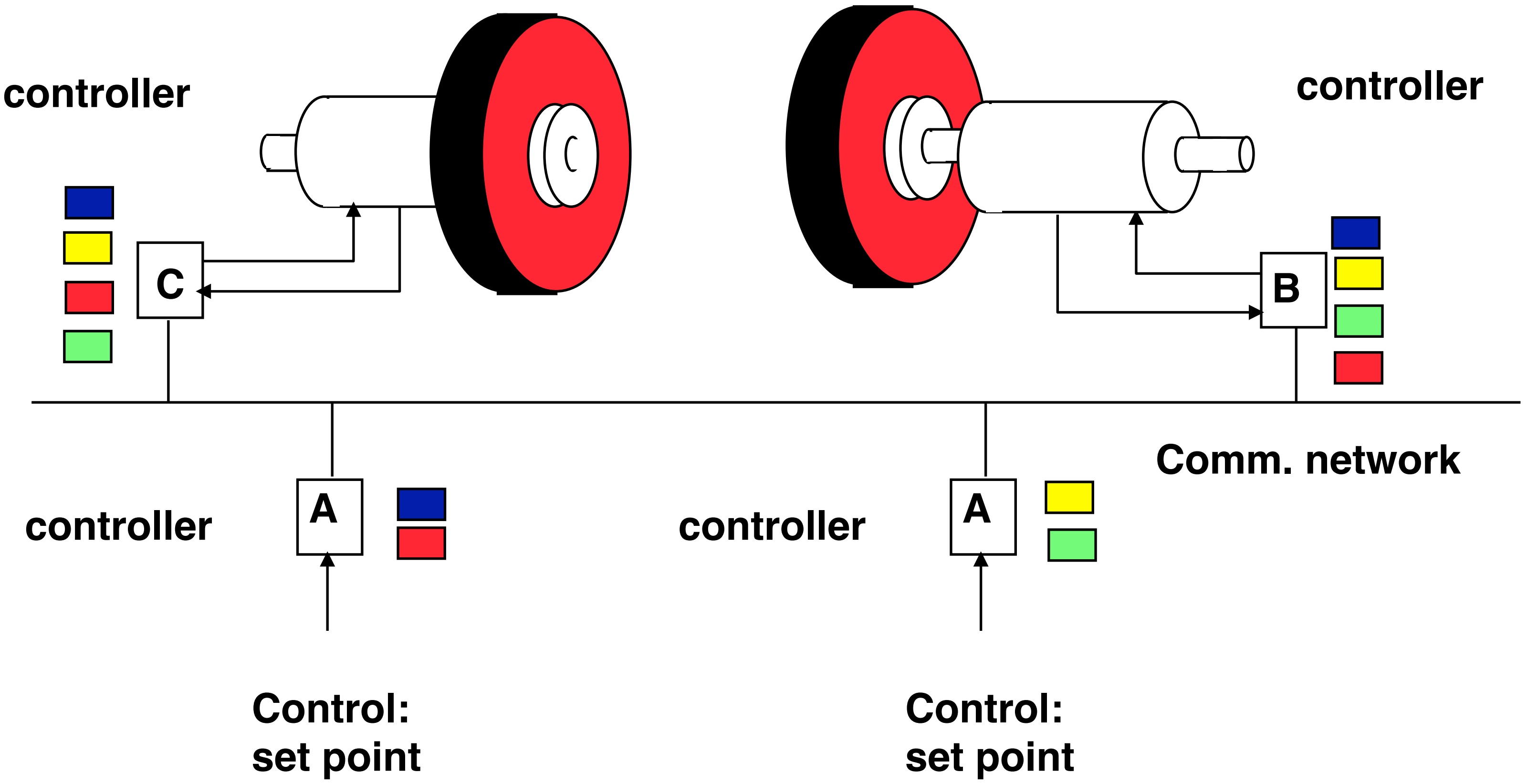
Order
in
Distributed Systems

Why Order?

- Determine the potential order of events.
  - Determine the cause-effect relationship (causality)in a distributed computation.
- Enforce an ordering policy, i.e. an a priori specified sequence of events.
  - Coordination of joint activities.

Order is important!



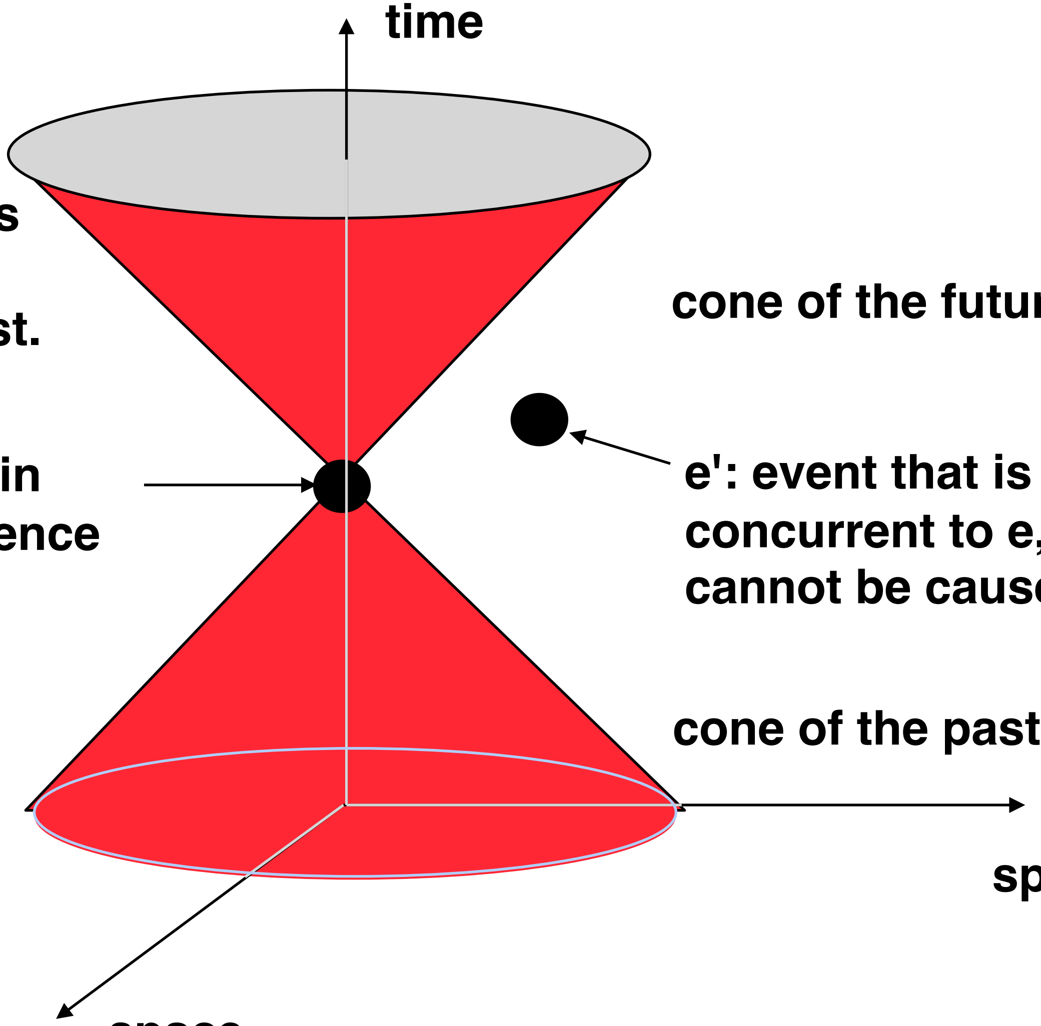
I.

What can be ordered?

In what way order is established in a distributed system?

What can be ordered?

e only has an impact on events in the future and only can be caused by events from the past.

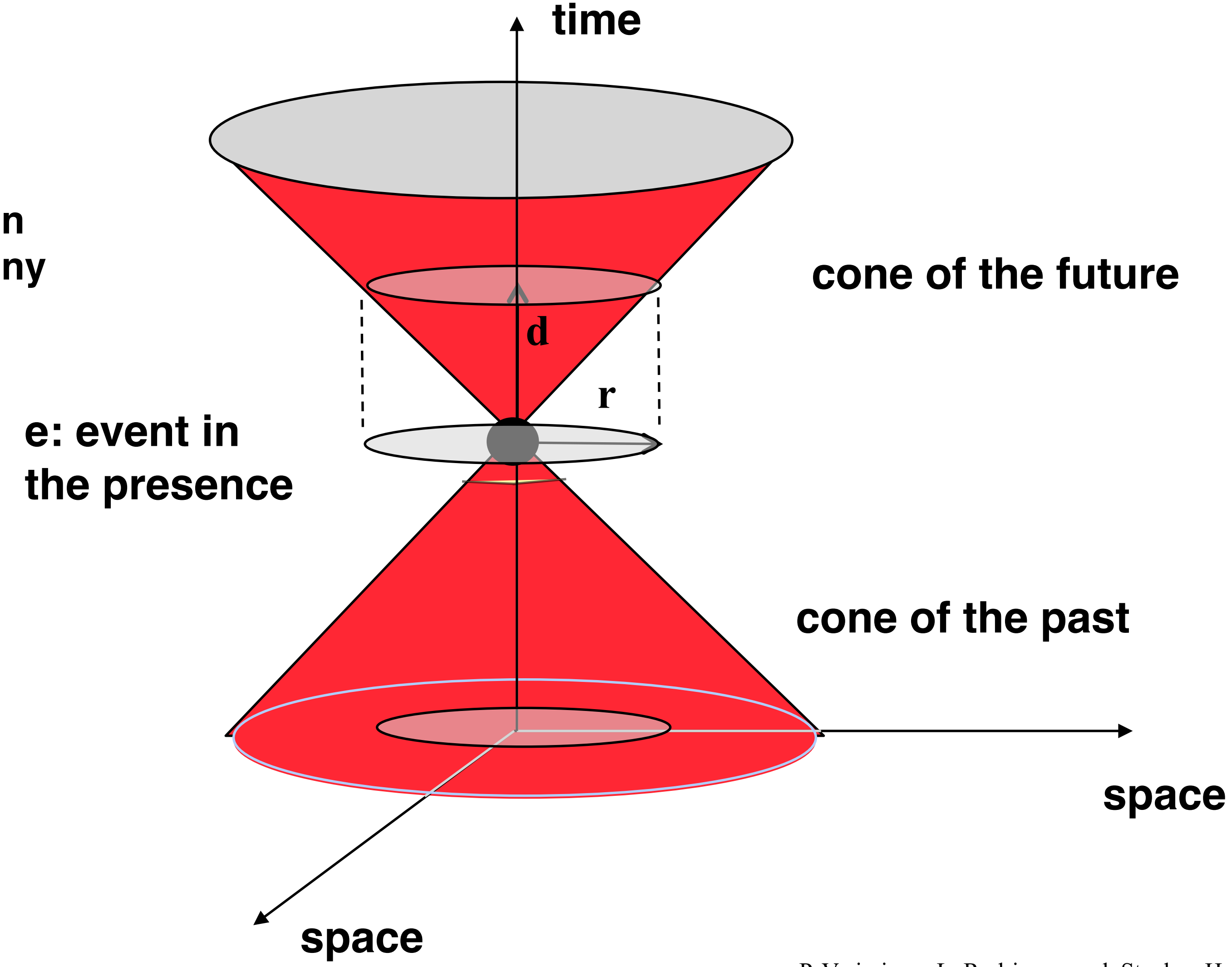


Can we (in principle) establish a total temporal order in a distributed system?

P. Verissimo , L. Rodrigues nach Stephen Hawking

What can be ordered?

r: max. spatial extension
d: max. delay to reach any
    point in space.



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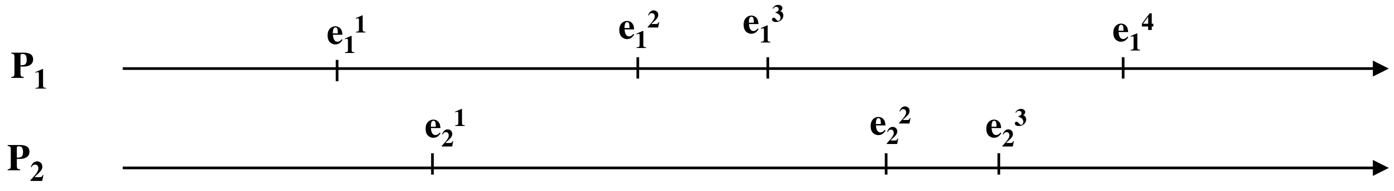
Computational Model

- A distributed computation is performed as the joint activity of local, sequential processes.

- The activity of a local sequential process is modelled as a sequence of events.



- An event either is local to a process, i.e. it causes an internal, local state change, or

- A computation includes the comminication with another process. This will be modelled by a send and a receive event.

- Messages are unambiguous single events, i.e. multiple messages with the same contents sent by the same process will be modelled as multiple individual events.

- All models of Data Sharing are abstracted as communication.

Computational Model

Def.:
The local history of proces p_i is a (possibly infinite) sequence of events $h_i = e_i^1\ e_i^2\ e_i^3\ \ldots .\ e_i^n \ldots$ (canonical enumeration). It defines a total order of local events.

Def.:
The global history is the set $H = h_1 \cup h_2 \cup h_3 \cup \ldots \ldots . \cup h_n$.

Note: The global history does not specify any relative time or order between the elements.

The Precedence Relation

Events in a system can be ordered according to their causal relationship in a cause-effect chain (happens before relation, Lamport 78)).

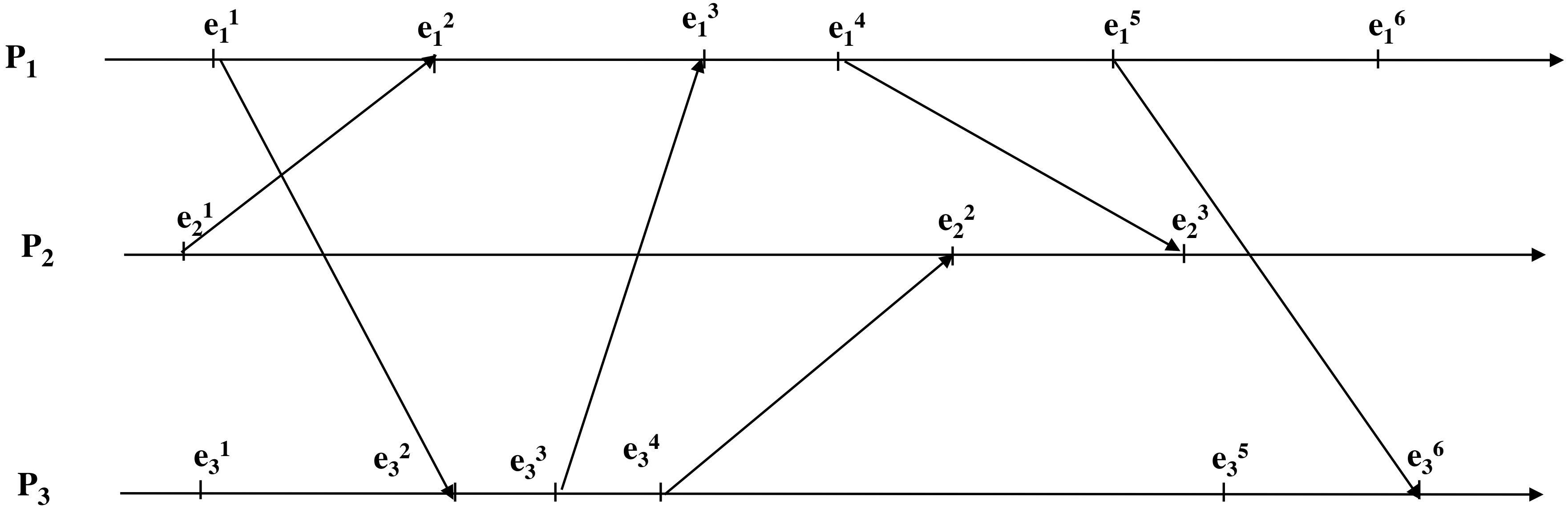
Def.: Precedence Relation →

1. for all e_i^k , $e_i^l \in h_i$, $k < l$: $e_i^k \rightarrow e_i^l$ (h_i is the "history" of process i) (local precedence)
2. If e_i = send (m) and e_j = receive (m) : $e_i \rightarrow e_j$
3. If e → e‘ and e‘ → e“ : e → e“ (transitivity)

For concurrent events no causal relationship can be specified, i.e. neither e → e‘ nor e‘ → e holds. Notation: e || e‘

A distributed computation can formally be seen as a partially ordered set defined by the tupel (H, →) where H is the combined History of all processes.

Time-Space Diagram



$e_1^2 \rightarrow e_3^6$

$e_3^1 \parallel e_1^2$

What is the meaning of consistency in a distributed system

?

**A system state, that can be established by any possible sequential execution of processes.
Causality must be preserved.**

Global States and Cuts

- **Local state:**
  z_i^k : local state of p_i after the execution of e_i^k

  z_i^0 : Initial state of p_i

- **Global state: $\Sigma = (z_1 , z_2 ,. . . . , z_n)$**

- **Def.: The *Cut C* of a distributed computation is the subset C of the global history H with:**

$$C = h_1^{C_1} \cup h_2^{C_2} \cup h_3^{C_3} \cup . . . . \cup h_n^{C_n} .$$

The tupel of integers $(c_1 , c_2 ,. . . . , c_n)$ represents the resepective index of the last event that has been considered for every process. The set of most recent events $\{ e_1^{C_1}, e_2^{C_2}, e_3^{C_3}. . . . . . ., e_n^{C_n} \}$ is called the *front* of the cut.

A global state $(z_1^{C_1}, z_2^{C_2}, z_3^{C_3}. . . . . . ., z_n^{C_n})$ is associated with every cut $(c_1 , c_2 ,. . . . , c_n)$.

Runs

Def.: *Run* of a distributed computation:

A *run* of a distributed computation is a totally ordered sequence R, that comprises all events of the global history and is compliant to every local history.

Consistency

1.) A *run* is consistent, if:

for all e, e‘ : (e $\in$ C) and (e‘ $\rightarrow$ e) $\Rightarrow$ e‘ $\in$ C

2.) A consistent (distributed) state is represented by a consistent cut.

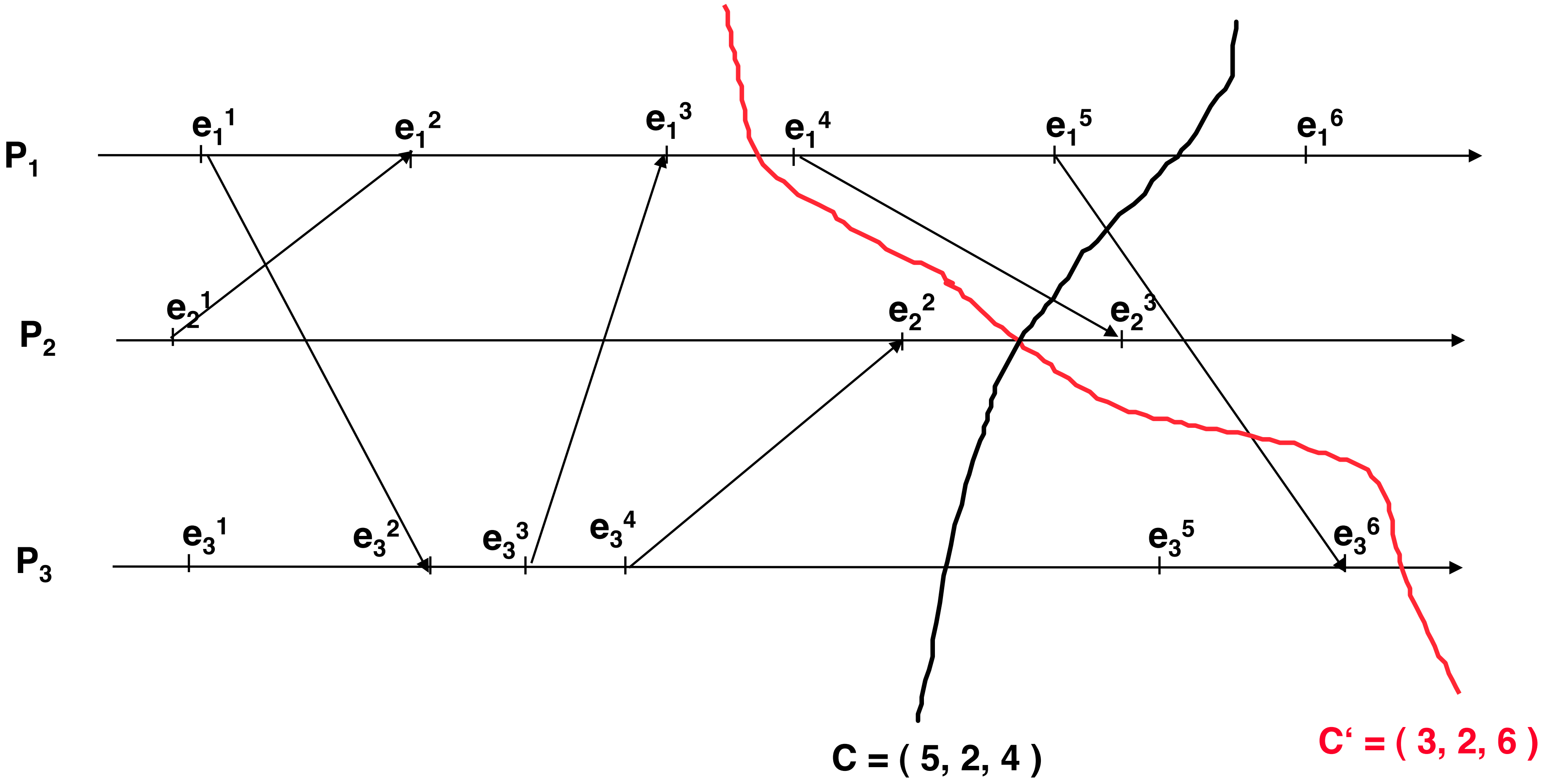
Analogies between sequential and distributed computations:

I. **The point in time of an event in a sequential computation is equivalent to:**
The *front* of a consistent cut in a distributed computation.

II. **e appears *before* or *after* a certain point in time of a sequential computation is equivalent to:**
e appears *before* or *after* a cut C, if the event is *left* or *right* of the *front* of C in a distributed computation.

The *run* of a distributed computation is consistent if for all events the following condition holds:
e $\rightarrow$ e‘ implies that e appears before e‘. (*R* defines a total order)

Runs, global states and cuts



Example: $R = (e_2^1, e_1^1, e_1^2, e_3^1, e_3^2, e_3^3, e_3^4, e_2^2, e_3^5, e_1^3, e_1^4, e_1^5, e_1^6, e_3^5, e_2^2, e_2^3, e_3^6)$

Ordering messages in Distributed Systems

How to order messages ?

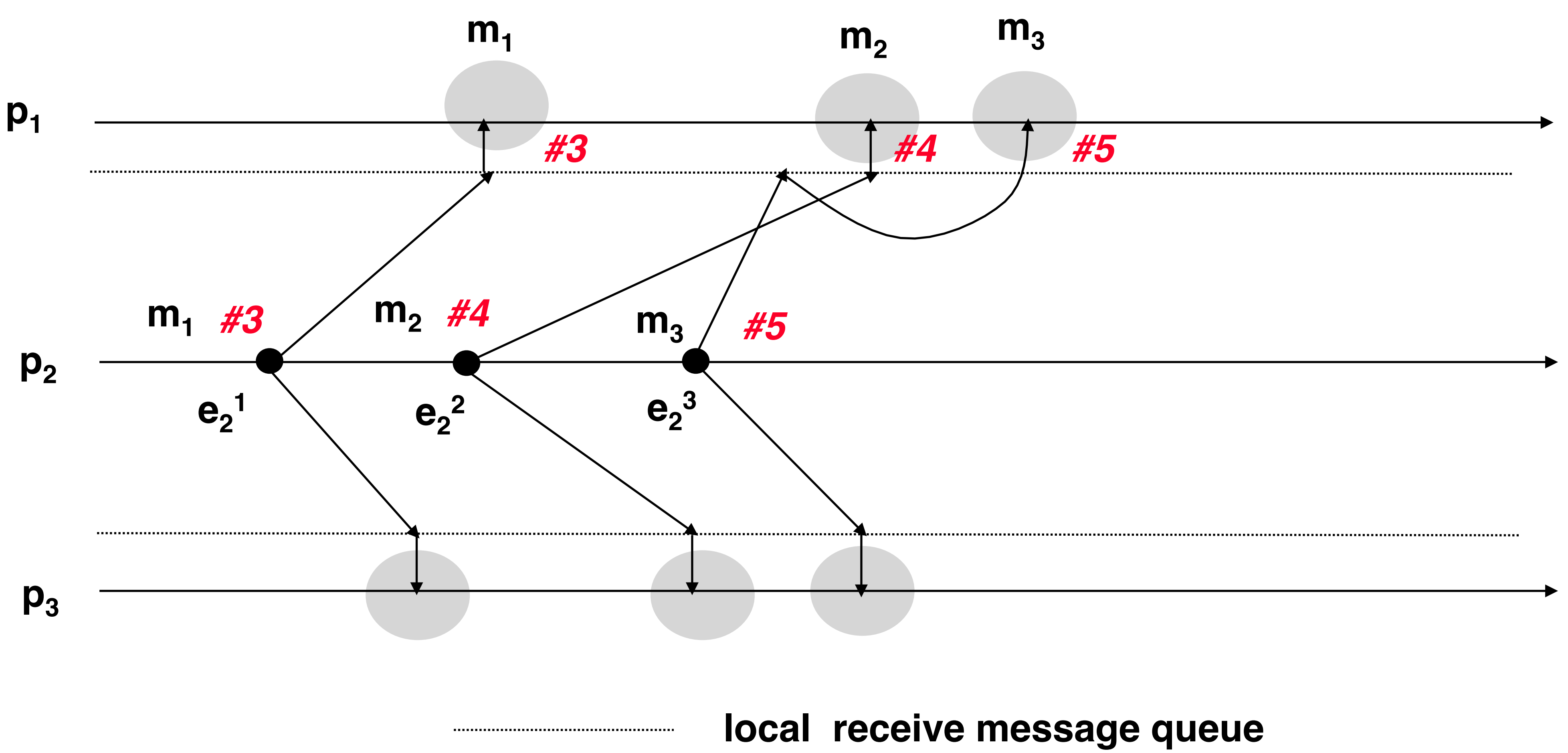
Temporal order: messages are ordered in a way that the message m_1 sent before message m_2 also will arrive before m_2.

FIFO ....

CAUSAL ....

TOTAL ....

FIFO-Receive order for pairs of processes



FIFO-order for pairs of processes

- **Idea:** **Receive process reorders the messages.**
- **Approach:** **Distinguish the *receiption* of the message at the node from the *delivery* to an application process**

***FIFO-delivery* :** $\mathbf{send_i(m) \rightarrow send_i(m') \Rightarrow deliver_j(m) \rightarrow deliver_j(m')}$

FIFO-D prevents a message from overtaking a message sent later between two processes.

FIFO-order for pairs of processes

Properties:

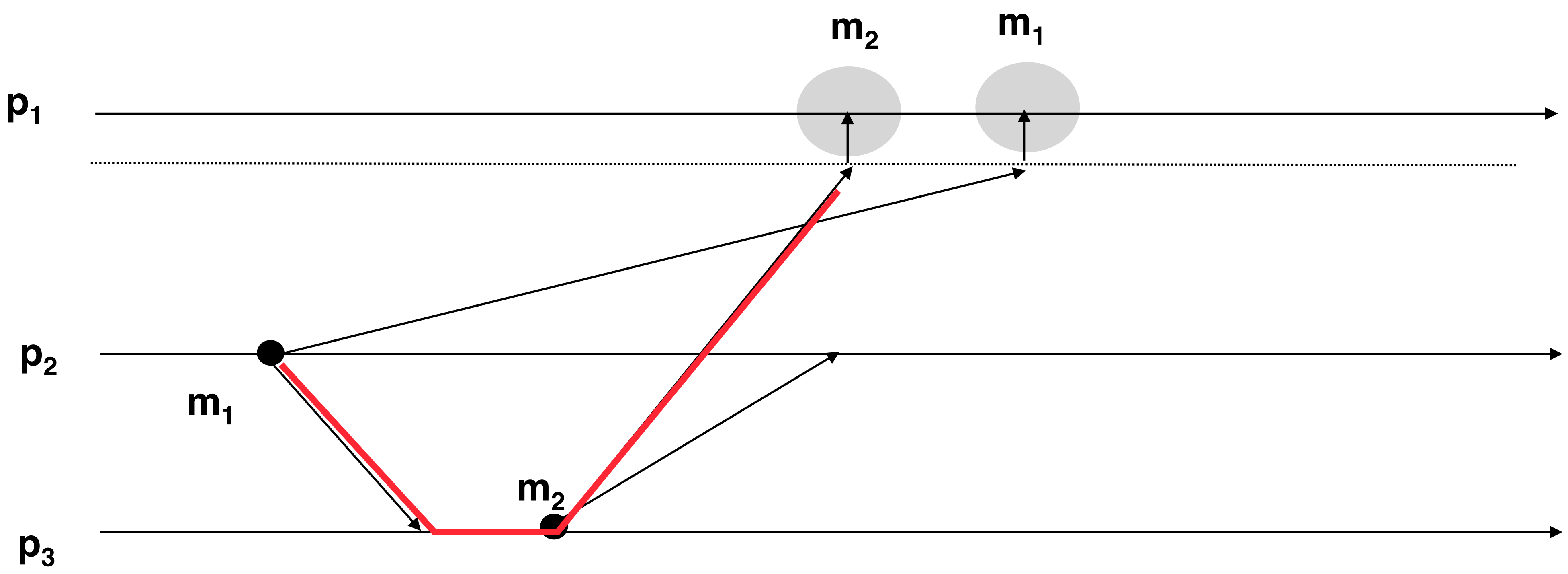
Overhead: Process needs to add a sequence number

FIFO-D is sufficient to guarantee that an observation complies to some run because FIFO-D maintains the order of local events.

BUT:

Because FIFO-D is defined between pairs of processes only it is not sufficient to guarantee that the observation corresponds to a consistent run !

FIFO-D is insufficient



The order of events which p_1 constructs based on the sequence of messages is inconsistent.

FIFO-D doesn't reflect causality for messages sent by different processes!

Causal Delivery

Causal Delivery:
For all messages m, m' and all processes p_i , p_j (send-prozesses) and p_k (receive-prozess) holds:

Causal-D **(CD):** $\mathrm{send}_i(m) \rightarrow \mathrm{send}_j(m') \Rightarrow \mathrm{deliver}_k(m) \rightarrow \mathrm{deliver}_k(m')$

CD maintains the global causal order of all messages in the system.

Causal Delivery

Events e und e’ may be causally dependent.

To realize causal delivery, we must be able to decide

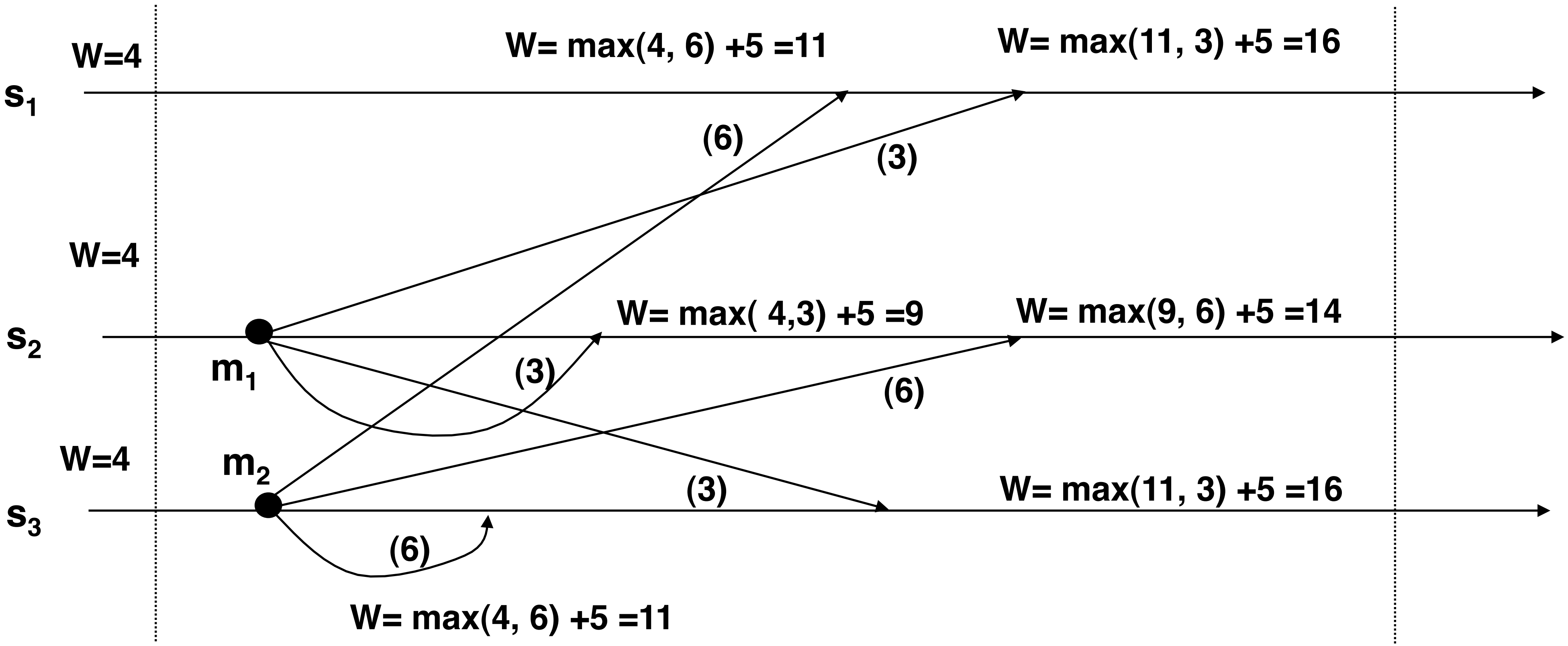
Is there any event e” with the property:

e → e“ → e‘

?

It is necessary to order the events along causal dependencies.
The temporal sequence of events only defines a potential causal relationship.
Note: Temporal order does not violate causal order.

Is causal order sufficient ?



Every sensor process s_i maintains a variable W that represents a global state e.g. the state of the environment. A new value is calculated from the old value and the messages from the other sensors W_t = max (W_{t-1} , sensor message) + 5

Requirement:

1. All nodes have the same order of messages
2. The order should reflect the causal relationships correctly.
3. Concurrent messages have an arbitrary order.

How to realize?

Total order

Goal: Observer, which orders all local events in a consisten global stream of events

⇒ produce a *totally ordered* event stream.

Intuitive solution:
Use global time.

Assumptions:

1. **All processes have access to a global clock and can take timestamps from that.**
2. **Communication latencies can be bounded by *d* .**

RC(e) is the value of the global clock when the event e occurs.
RC(e) is added as timestamp TS to the message.

Delivery rule:

DR 1 : At time t deliver all received messages in ascending order of the timestamps TS with TS = t - d.

Why is global consistency ensured by DR 1?

Condition I:
The latency of messages is bound by *d*. Therefore, at time t all messages sent before t-*d* have been received. No message sent earlier than t-d will ever be received after t.

Condition II:
The observation is consistent iff the clock condition : $e \rightarrow e' \Rightarrow RC(e) < RC(e')$ holds.
This condition is ensured by the global time.

Disadvantage: **Availability of global time.**
Question: **Can consistency of ordering be achieved without physical time?**

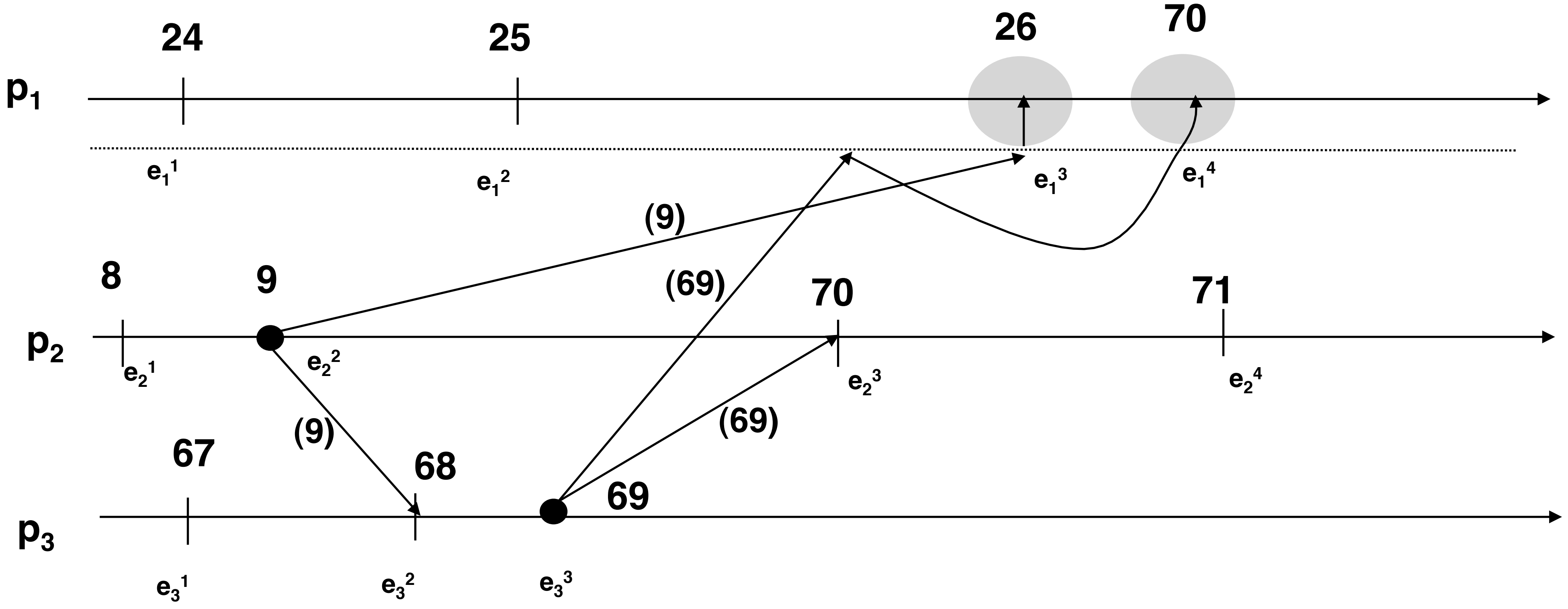
Logic Clocks (Lamport 1978)

Basic Idea: To achieve a consistent order of messages, we only have to consider the causal relationships. Concurrent messages can be ordered arbitrarily BUT everywhere in the same order.

i.e.

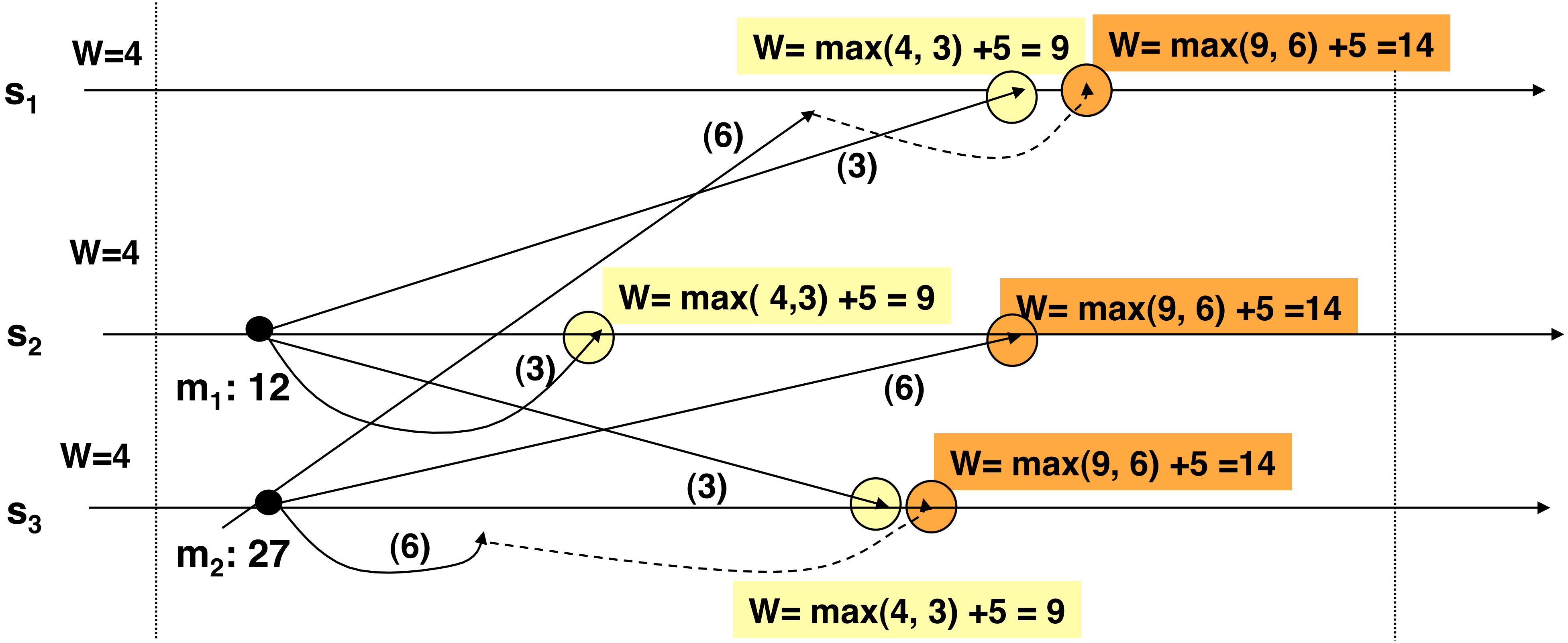
The order based on ascending logical time must correspond to the causal order.

Total Order



24
25
26
70
p1
e1 1
e1 2
e1 3
e1 4
(9)
8
9
(69)
70
71
p2
e2 1
e2 2
e2 3
e2 4
(9)
(69)
67
68
69
p3
e3 1
e3 2
e3 3

Total Order


W=4
s1
W= max(4, 3) +5 = 9
W= max(9, 6) +5 =14
(6)
(3)
W=4
W= max(4,3) +5 = 9
W= max(9, 6) +5 =14
s2
m1: 12
(3)
(6)
W=4
W= max(9, 6) +5 =14
(3)
s3
m2: 27
(6)
W= max(4, 3) +5 = 9

Logic Clocks

- Every process maintains a variable LC that represents the individual logical clock. LC maps local events on positive intergers.

- LC(e_i): logical clock value of process p_i, when event e_i is generated.

- Every message m that is sent carries the timestamp TS(m), which represents the logical clock value of the sending process.

- Initialization: Before any event is generated, all logical clocks will be reset to "0".

- The following update rule defines the logical clock modification of process p_i when event e_i occurs:

$$LC(e_i) := \begin{cases} LC + 1 & \text{if } e_i \text{ is a local event or a send event} \\ \max\{LC, TS(m)\} + 1 & \text{if } e_i \text{ is a receive event} \end{cases}$$

Properties of Logic Clocks

- **Local clocks always produce increasing values**

- **Logic clock values are increasing with respect to causal order**

- **Logic clocks satisfy the condition : $e \rightarrow e' \quad \Rightarrow LC(e) < LC(e')$.**

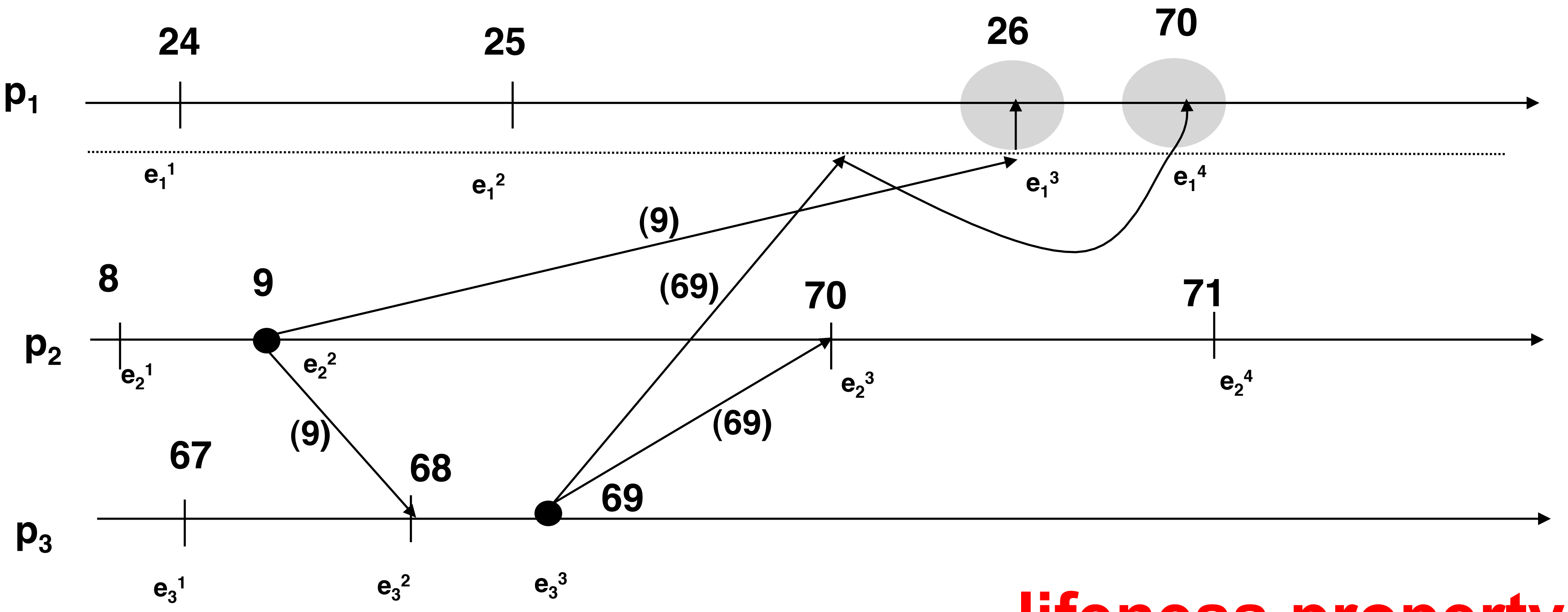
  This is called the weak Clock Condition because: $LC(e) < LC(e') \quad \not\Rightarrow \quad e \rightarrow e'$

Question:

Are logic clocks sufficient to guarantee consistent observations?

Total order is correctly established by logic clocks

safety property



lifeness property

BUT: in an asynchronous system it is impossible to determine when a message may be delivered.

Gap-Detection Property (GDP)

Given the events e und e’ with clock values LC(e) and LC(e’).
The condition LC(e) < LC(e’) holds.

GDP denotes the ability to decide whether there exist an event e’’ which satisfies LC(e) < LC(e’’) < LC(e’)

GDP is needed to guarantee lifeness.

Problem: Find an algorithm with the following properties:

1. All events are totally ordered
2. On the basis of receive events it can be decided when a message can be delivered

Note: Real-time clocks don't solve the problem!

Gap-Detection Property (GDP)

Matrix Clocks
Vector Clocks
have the GAP detection property

Synchronous protocols solve the GAP detection problem.

Synchronous Systems

The communication system has a known and bounded maximal message delay *d*.

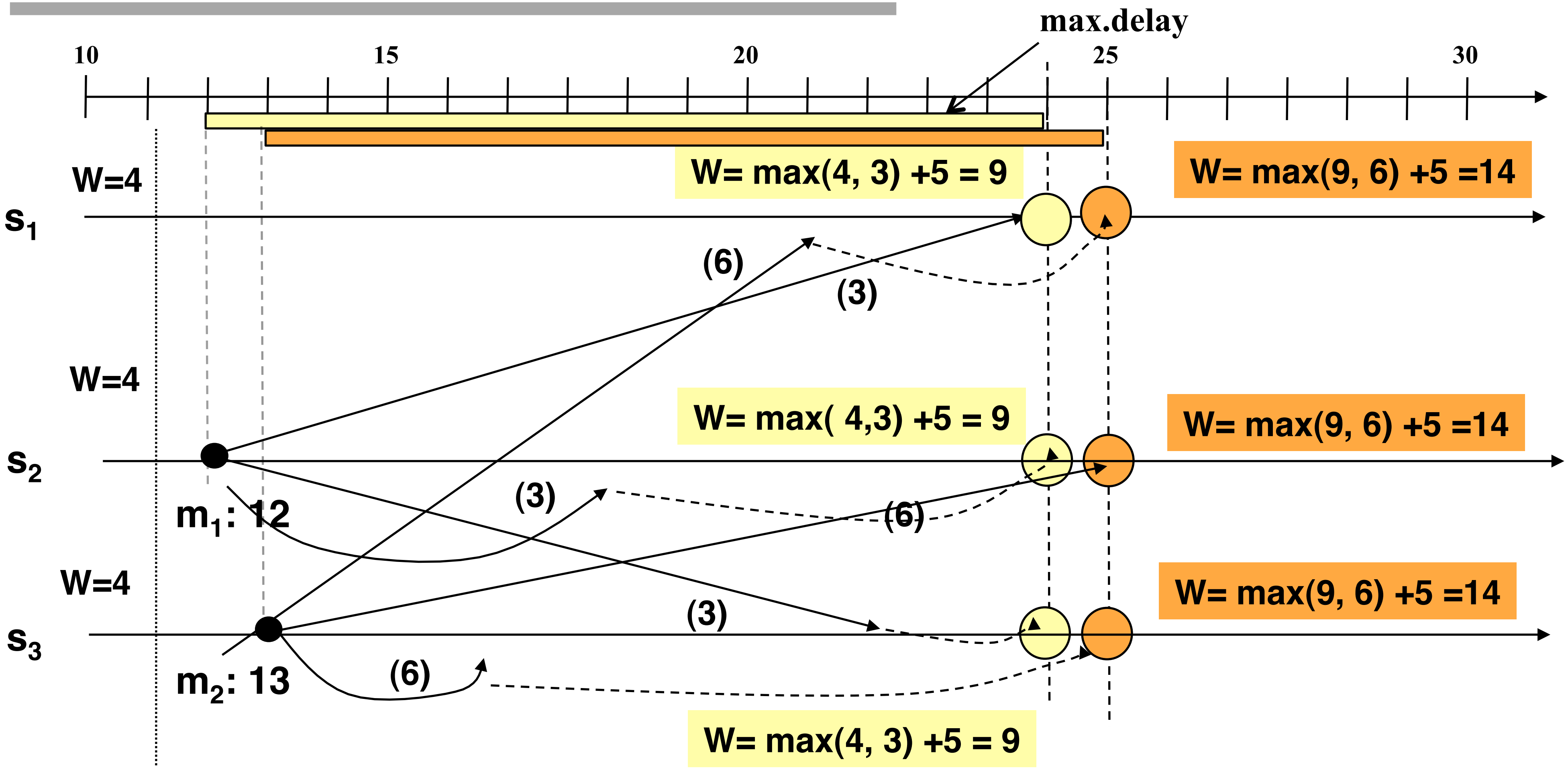
All processes have access to a global real-time clock (RC).

RC(e) is the value of the global clock when event “e” occurs.
RC(e) is added as timestamp TS to the message

Delivery rule:

At time t deliver all messages in ascending order with TS = t - d.

Total Order in a Synchronuous System



Temporal order

A message m_1 is temporally preceding a message m_2 if m_1 is sent at least δ before m_2 , i.e. :

$$t(send(m_1)) - t(send(m_2)) > \delta$$

According to this definition, a protocol that delivers messages in temporal order also guarantees causal order.

Synchrony Metrics

Problem # 1:
How big is the max. difference of message propagation of ONE message?

→ Tightness

Problem #2:
How big is the max. difference of message propagation of DIFFERENT messages?

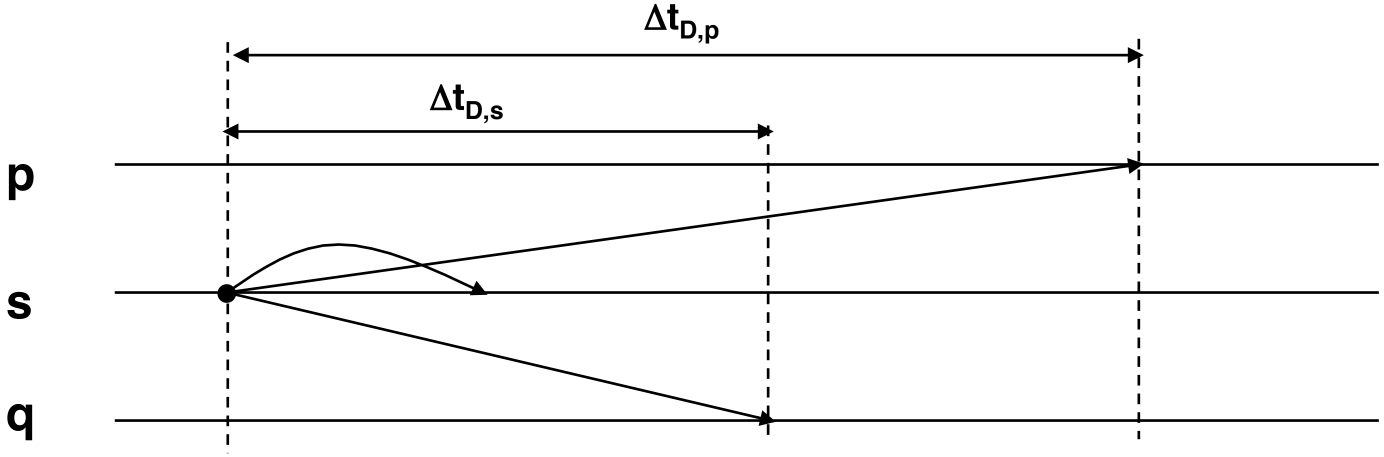
→ Steadiness

Synchrony Metrics

Definition: Delivery time of a message

$\Delta t_{D,p}$ = t (deliver $_{p}$(m))- t (send(m))

$\Delta t_{D,p}$: Interval between the send event of message m its delivery at prozess p



Synchrony Metrics

Tightness τ

Definition: Tightness

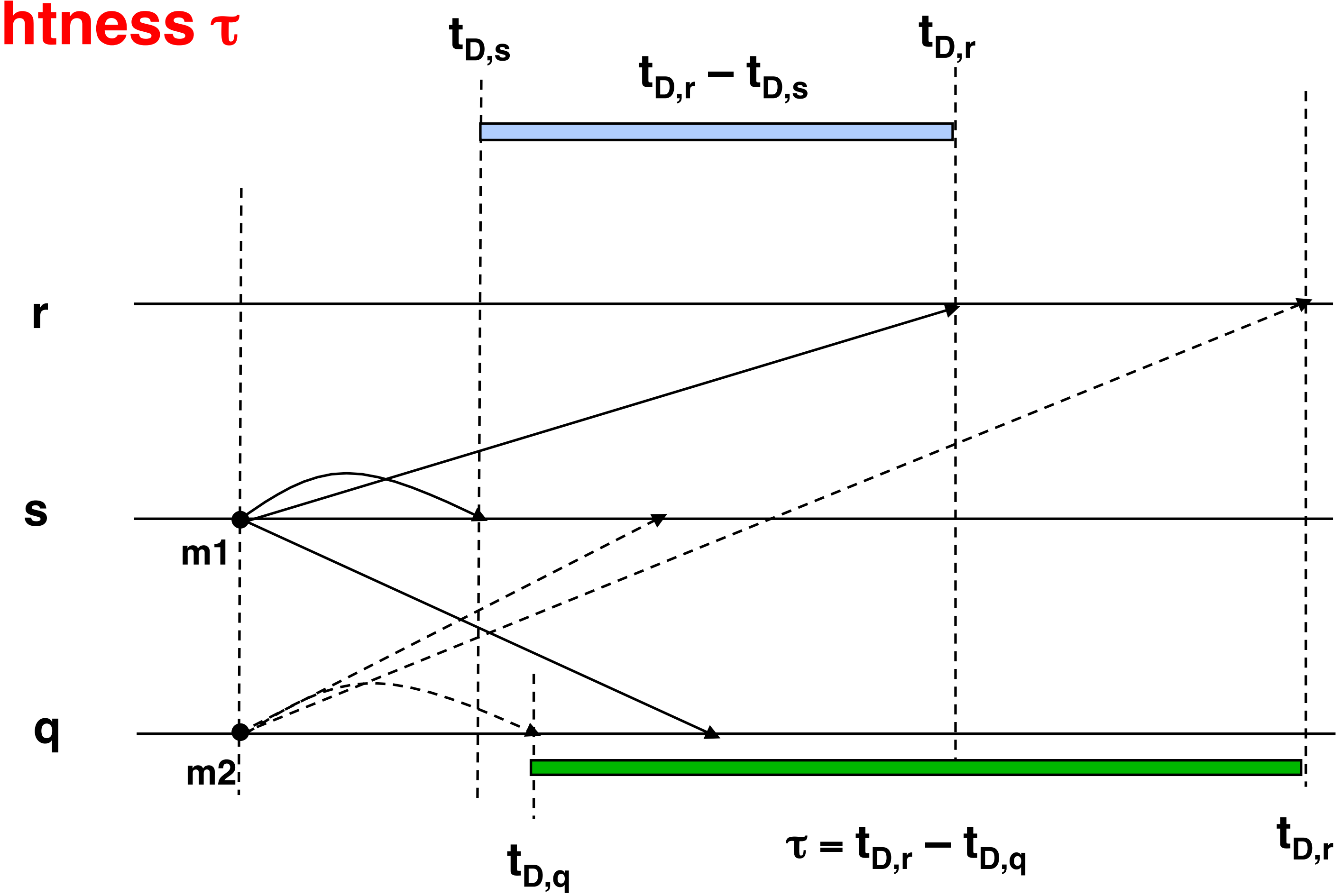
$\tau = \max_{m,p,q} (t_{D,p} - t_{D,q})$

for every message m, τ is the maximal difference of transmission times that occurs for arbitrary receivers p and q.

Tightness is a measure for the difference of transmission times of ONE message to DIFFERENT nodes.

Synchrony Metrics

Tightness τ



Synchrony Metrics

Steadiness σ

Definition: Steadyness

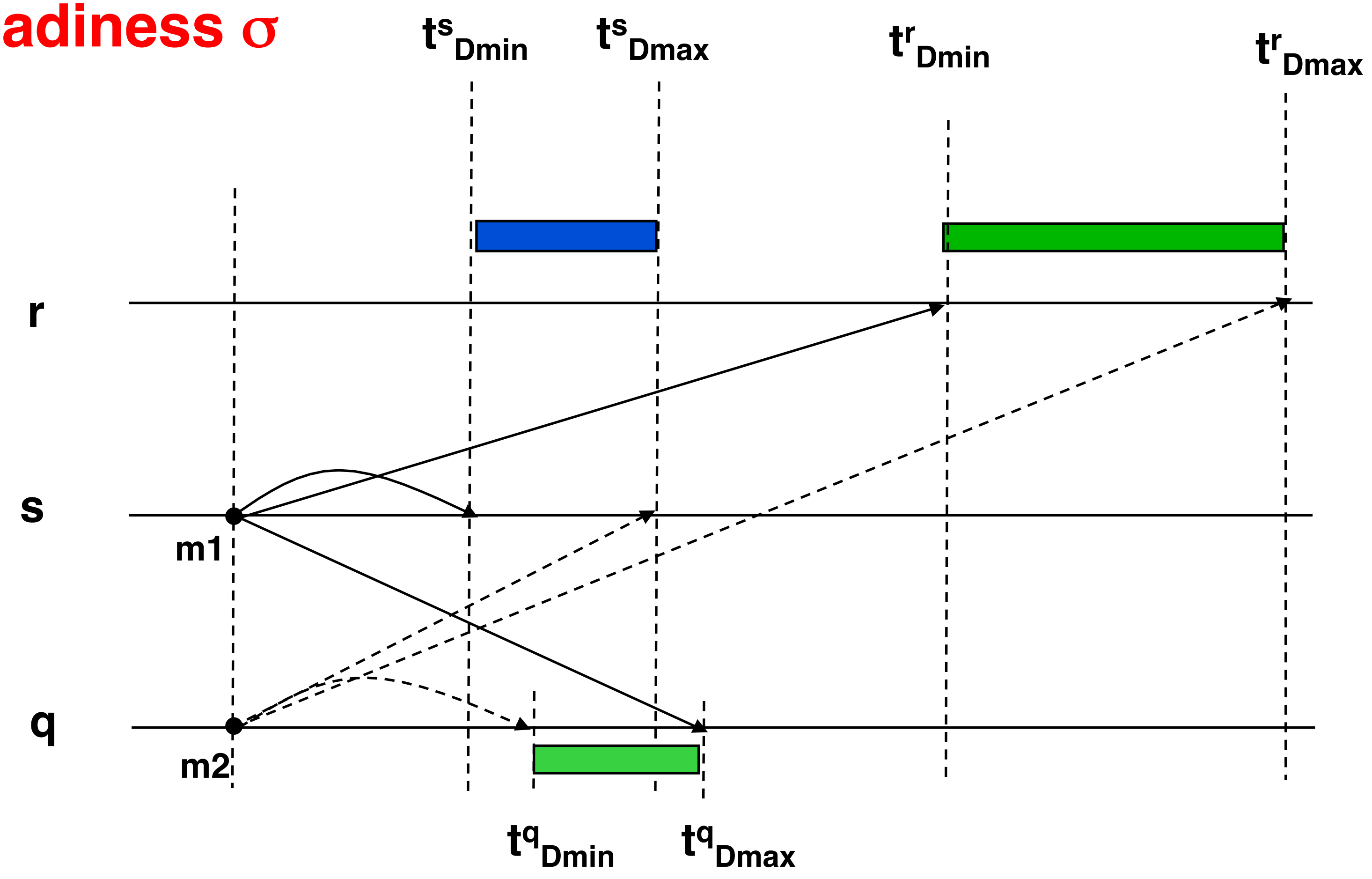
$\sigma = \max_p (t^p_{Dmax} - t^p_{Dmin})$

σ is the maximal difference that can be observed between the maximum t^p_{Dmax} and the minimum t^p_{Dmin} delivery times of different message (to some process p).

Steadiness measures the maximal difference of delivery times of DIFFERENT mesages to the same process.

Synchrony Metrics

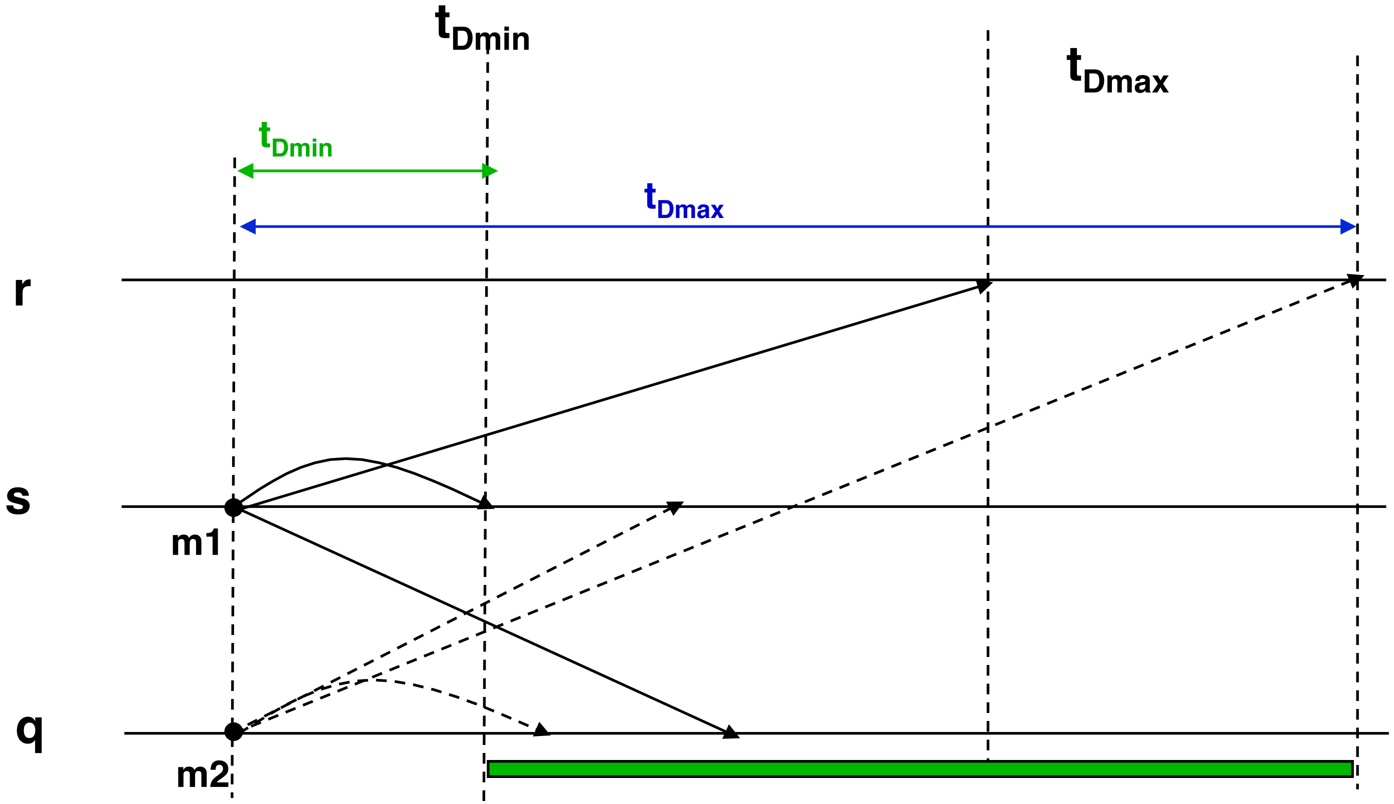
Steadiness σ



$$\sigma = \max_p \left(t^p_{Dmax} - t^p_{Dmin}\right)$$

Synchrony Metrics

Temporal uncertainty ε

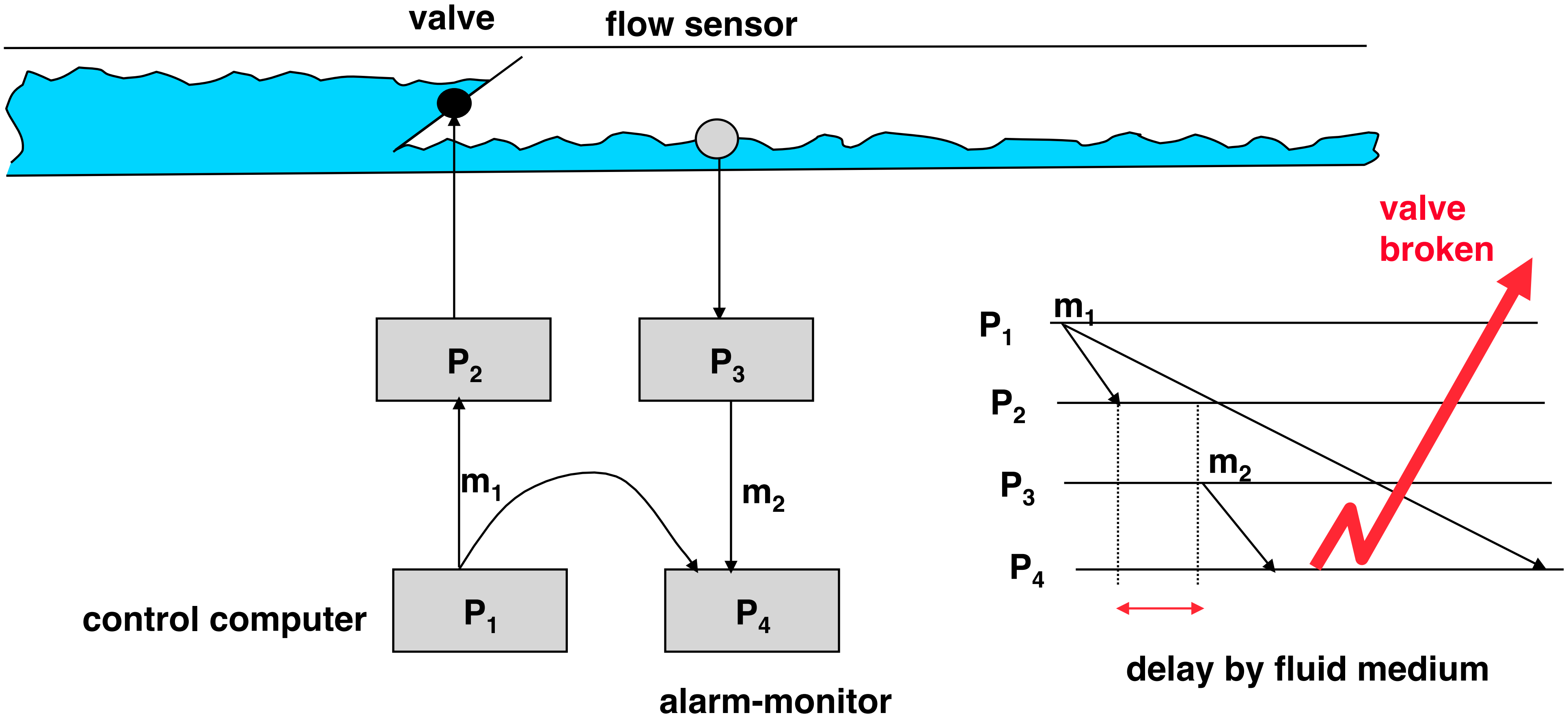


$$\varepsilon = \max_{p,q} \left(t^{p}_{Dmax} - t^{q}_{Dmin} \right)$$

Temporal uncertainty describes the absolute max difference between the minimal delivery time and the maximal delivery time of all messages.

What is ordered by logic order?

The problem of hidden physical channels



valve
flow sensor
P_2
P_3
m_1
m_2
control computer
P_1
P_4
alarm-monitor
valve broken
P_1
m_1
P_2
m_2
P_3
P_4
delay by fluid medium